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Equivalent Circuits of Linear Active Four-Terminal Networks*

By

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Introduction

THE art of equivalent network representation has grown very considerably since its inception by Dr. G. A. Campbell. In his paper "Cissoidal Oscillations" which was published in 1911 he proved that any passive network made up of a finite number of invariable elements and having one pair of input terminals and one pair of output terminals is externally equivalent to an unsymmetrical T or Π network. From this modest beginning the field of applications of the equivalent circuit concept has steadily expanded so that by now the whole field of linear passive circuit theory has been subjected to equivalent circuit interpretation.

With the advent of the thermionic vacuum tube amplifier, linear active network theory had to be considered and almost immediately the attempt was made to obtain an equivalent circuit whose performance would depict the linear characteristic of the tube. The equivalent circuit art has also, in recent years, been used to describe the performance of certain classes of non-linear devices, such as mixers, and further applications in this field will no doubt be made.

Equivalent circuit concepts have played an important part in electrical

* This paper appears substantially as it was originally prepared some years ago as a technical memorandum for internal distribution within Bell Telephone Laboratories. Its publication is rendered timely by the applicability to the recently announced transistor devices. Present experience indicates, in fact, that the configuration of Fig. 13 furnishes the most useful equivalent four-pole network for the transistor.

Mr. J. A. Morton has called my attention to an early paper in this field by Strecker and Feldtkeller (E.N.T. Vol. 6, page 93, 1929) in which the general theory of active networks has been well developed. However, the early state of the then prevailing art prevented the full demonstration of the power of the method and of course precluded the possibility of application to modern devices. This paper enlarges the general theory and makes logical applications of the method to modern devices. Since the appearance of the Strecker-Feldtkeller paper several other papers touching upon this subject have appeared. However, no attempt at giving a complete bibliography will be made, except to call attention to the contributions of Prof. M. J. O. Strutt, who also has adopted the four-pole point of view.

The I.R.E. Electron Tube Committee has adopted the four-pole viewpoint and has proposed methods of tests for experimentally determining the four-pole parameters of electron tubes. This material will be published in the new I.R.E. standards on electron tubes.

engineering, particularly in communication engineering. One might almost say that a problem is not solved unless an equivalent circuit has been found whose performance will exhibit some of the characteristics of the actual problem. This need for circuit concepts reflects a desire to make the phenomena more alive and subject to physical interpretation, for it is true that equivalent circuit concepts have greatly contributed towards physical interpretation of analytical expressions. A problem may very well be studied without recourse to the equivalent circuit concept and a correct answer obtained, yet the development of an equivalent circuit adds greatly to the complete interpretation of the physical phenomena.

In this paper we shall be concerned only with linear a-c amplifier operation where the term *linear* indicates that the analytical expressions connecting currents and voltages are linear, that is, involve only the first power of any instantaneous current or its derivative. We shall further restrict our attention to the usual mode of four-terminal amplifier operation in which one pair of terminals is associated with the signal to be amplified and the otber pair with the amplified signal. The equivalent a-c circuit of such a transducer will require the development and interpretation of the linear relations connecting the a-c currents and voltages at the input terminals with corresponding quantities at the output terminals. At this point we can logically postulate that the important formal difference between an active and a passive four-pole lies in that the law of reciprocity no longer can be assumed to hold for the active network. Since passive four-poles require three independent parameters for their complete specification the active four-pole will require at least one additional parameter.

In the practical applications we shall be principally concerned with the various triode four-pole connections. A short review of the various stages involved in deriving the newer forms of equivalent triode circuits will therefore be considered prior to the introduction of generalized concepts. Such a review is in the main concerned with the effect of frequency upon the early forms of the equivalent triode circuit.

In the review we shall confine ourselves to the usual grounded cathode mode of operation since it is only in recent times that grounded grid and grounded plate operation have come into use. This distinction is, however, not necessary and is introduced solely for simplicity.

The conventional notation as well as the positive current directions are indicated on Fig. 1 for a general four-pole N. It is assumed here that terminals 1 are tbe available input and terminals 2 the available output terminals. This choice of current direction appears to the writer to be the most convenient to use when the four-pole is energized at the input terminals 1 only.

Consider, now, a grounded triode operated at such a low frequency that

all displacement or capacity currents can be disregarded, and let it first be supposed that the grid is negatively polarized with respect to the cathode so that grid current is absent. This represents the most primitive form of operation, governed by a law which has been termed "The equivalent-plate-circuit Theorem."[1] With reference to the assumed current direction this theorem may be expressed by saying that the application of the voltage V_1 to the grid is equivalent, so far as phenomena in the plate circuit are concerned, to the application of the voltage $-\mu V_1$ in series with a resistance r_p, where μ is the amplification factor and r_p the internal plate resistance. The

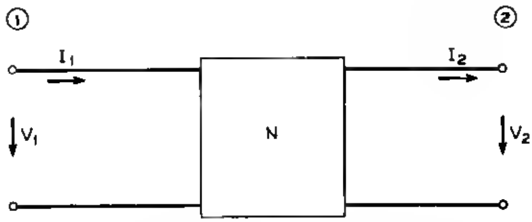


Fig. 1—Current-voltage relations for a general four-pole.

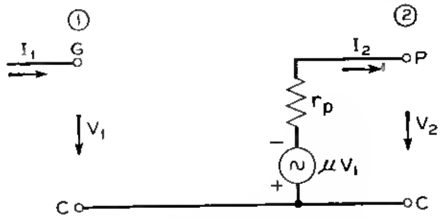


Fig. 2—Equivalent circuit of a negative grid triode at low frequencies.

equivalent circuit for this mode of operation is thus as shown in Fig. 2, where the input terminals 1 are represented by the grid G and the cathode C and the output terminals by the anode P and the cathode C. In terms of analysis the circuit is described by the two equations

$$\left.\begin{aligned} I_1 &= 0 \\ -\mu V_1 &= I_2 r_p + V_2 \end{aligned}\right\} \tag{1}$$

Equations (1) and their associated circuit Fig. 2 are expressions of the fact that in any closed loop the voltages must be in equilibrium.

By a slight rearrangement of (1) a network representation based on current

[1] Chaffee, "Theory of Thermionic Vacuum Tubes," page 192.

equilibrium may be obtained. For this purpose (1) is written as

$$\left.\begin{aligned} I_1 &= 0 \\ I_2 &= -\frac{\mu}{r_p} V_1 - \frac{1}{r_p} V_2 \end{aligned}\right\} \tag{2}$$

The corresponding network representation is as shown in Fig. 3, where the energizing source in the plate circuit consists of a constant current generator of strength $-\frac{\mu}{r_p} V_1$ impressed across the output terminals 2.

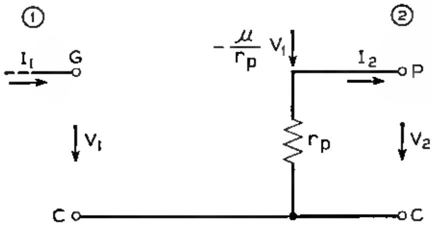


Fig. 3—Equivalent circuit of a negative grid triode at low frequencies.

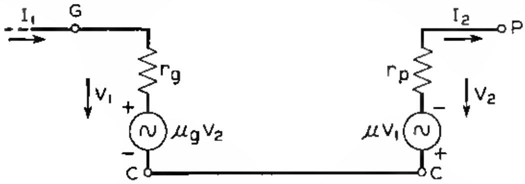


Fig. 4—Equivalent circuit of a positive grid triode at low frequencies.

Let us now consider a further step and assume that the grid is positive so that grid current is also flowing. In regard to the grid circuit there is a theorem called the "Equivalent-grid-circuit Theorem" which is exactly similar to the corresponding plate circuit theorem. The theorem says that the a-c grid current can be calculated by assuming that an e.m.f. $\mu_g V_2$ acts in series with a resistance r_g where μ_g is the reflex factor and r_g the internal grid resistance. In symbols and by using the notation of Fig. 1 this is expressed by:

$$V_1 = \mu_g V_2 + I_1 r_g \tag{3}$$

By combining the two theorems the equivalent circuit of Fig. 4 results. Again by writing (3) as

$$I_1 = \frac{1}{r_g} V_1 - \frac{\mu_g}{r_g} V_2 \tag{4}$$

a corresponding network based upon current equilibrium may be found.

It is well to note that this general low-frequency case is governed by the two simultaneous equations

$$\left.\begin{aligned} I_1 &= \frac{1}{r_g} V_1 - \frac{\mu_g}{r_g} V_2 \\ I_2 &= -\frac{\mu}{r_p} V_1 - \frac{1}{r_p} V_2 \end{aligned}\right\} \tag{5}$$

which are of tbe general form

$$\left.\begin{aligned} I_1 &= \beta_{11} V_1 + \beta_{12} V_2 \\ I_2 &= \beta_{21} V_1 + \beta_{22} V_2 \end{aligned}\right\} \tag{6}$$

where

$$\left.\begin{aligned} \beta_{11} &= \frac{1}{r_g} \quad \beta_{12} = -\frac{\mu_g}{r_g} \\ \beta_{21} &= -\frac{\mu}{r_p} \beta_{22} = -\frac{1}{r_p} \end{aligned}\right\} \tag{7}$$

The network of Fig. 4 is thus a possible form of circuit interpretation of (5) or (6). It may also be observed that (6) represents at least formally the most general formulation of the linear active four-pole so that even at very low frequencies the general four-pole point of view might be useful.

Several observations may now be made. In the first place it should be noted that these networks are not based on any study of the internal action of the tube, but rather on the purely formal mathematical process of differentiating the two functional relations which express the broad fact that plate and grid currents are some unspecified linear continuous functions of the grid and plate potentials in the neighborhood of the operating point.

In the second place it may be observed that the network of Fig. 4 represents in a sense two separate networks interacting with each other by means of voltage or current generators. This method of equivalent circuit representation is the result of separate interpretation of the equivalent plate and grid circuit theorems. As a corollary it follows that such a four-pole equivalence involves at least two generators within the network in order to take the effect of interaction into account.

We may say that the networks discussed were satisfactory so long as the frequency was low enough to allow displacement currents to be disregarded. With the operation of circuits at higher frequencies (up to the order of 10^6 cps, say) it became necessary to take the internal tube capacitances into account. This was done by the superposition of a capacity network as shown in Fig. 5. It is interesting as well as instructive to formulate this network transi-

tion in analytical terms. The transition rests upon the physical fact that the total current entering or leaving an electrode is the sum of conduction and

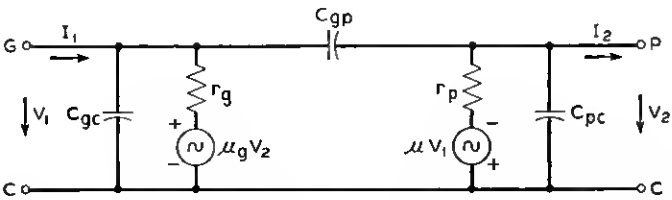


Fig. 5—Equivalent circuit of a positive grid triode at moderately low frequencies.

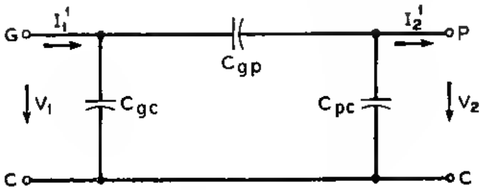


Fig. 6—Parasitic triode network.

displacement current. The network for the displacement currents is passive and is shown on Fig. 6. For this network:

$$\left.\begin{aligned} I_1' &= \beta_{11}'V_1 + \beta_{12}'V_2 \\ I_2' &= -\beta_{12}'V_1 + \beta_{22}'V_2 \end{aligned}\right\} \quad (8)$$

where I_1' is input and I_2' the output displacement current. The coefficients appearing have the values

$$\left.\begin{aligned} \beta_{11}' &= i\omega(C_{gc} + C_{gp}) \\ \beta_{12}' &= -i\omega C_{gp} \\ \beta_{22}' &= -i\omega(C_{pc} + C_{gp}) \end{aligned}\right\} \quad (9)$$

The potentials appearing in (8) are the same as those which occur in the conduction current equations (5), this being the physical condition which also must be satisfied. By adding (5) and (8) and by letting I_1 and I_2 mean total currents we get:

$$\left.\begin{aligned} I_1 &= \left[\frac{1}{r_g} + i\omega(C_{gc} + C_{gp})\right] V_1 - \left[\frac{\mu_g}{r_g} + i\omega\, C_{gp}\right] V_2 \\ I_2 &= \left[-\frac{\mu}{r_p} + i\omega C_{gp}\right] V_2 - \left[\frac{1}{r_p} + i\omega(C_{pc} + C_{gp})\right] V_2 \end{aligned}\right\} \quad (10)$$

These equations are again of the form

$$\left.\begin{aligned} I_1 &= \beta_{11} V_1 + \beta_{12} V_2 \\ I_2 &= \beta_{21} V_1 + \beta_{22} V_2 \end{aligned}\right\} \tag{11}$$

where now

$$\left.\begin{aligned} \beta_{11} &= \frac{1}{r_g} + i\omega(C_{gc} + C_{gp}) \\ \beta_{12} &= -\left[\frac{\mu_g}{r_g} + i\omega C_{gp}\right] \\ \beta_{21} &= -\frac{\mu}{r_p} + i\omega C_{gp} \\ \beta_{22} &= -\left[\frac{i}{r_p} + i\omega(C_{pc} + C_{gp})\right] \end{aligned}\right\} \tag{12}$$

The network interpretation of Fig. 5 is consistent with (10) and so does, in fact, constitute a possible network representation.

From (12) the relation

$$\beta_{21} + \beta_{12} = -\left[\frac{\mu}{r_g} + \frac{\mu_g}{r_g}\right] \tag{13}$$

is obtained. The thing to emphasize is that this sum is independent of the passive feedback admittance and that it is a constant independent of frequency within the range considered. It is, moreover, a quantity which only is correlated with the conduction currents in the tube. For lack of a better name the sum (13) will be referred to as "the effective transconductance." It will play an important role in the later discussion.

As still higher frequencies (above 10^7 cps) were employed it necame necessary to take into account lead effects, usually in the form of self and mutual inductances, and by incorporating them in the network of Fig. 5 a slightly more involved circuit was obtained. This more general network is still determined by equations of the form (11), for adding the new network elements merely means that linear transformations are applied to the potentials V_1 and V_2 with the result that a new set of β-coefficients is obtained; the new set being merely of somewhat more complicated form than the first.

With the utilization of frequencies so high that the electron transit times became comparable with the period of the applied signal, further complications arose and the equivalent network idea was put to a severe test. In

this development we may distinguish between two methods of approacb. In one the attempt was made to modify the low-frequency network of Fig. 5 to include transit time effects to a first order of approximation.[2] The second approach differed in that attention was directed only toward the electron stream itself, while the circuit elements connecting the stream with the physically available terminals were grouped together with the external circuit elements.[3] The latter approach represented a particularly useful one from a physical point of view and it also extended the use of basic circuit elements to include the general diode impedance as a new circuit element complete in itself. However, even in this latest approach the four-pole point of view was not adopted, with consequent loss of generality and unity in viewpoint. Moreover, the fact that only the electron stream itself was considered caused some confusion.

Witb this brief review of the development of equivalent circuit representation of vacuum tube amplifiers in mind we turn now to tbe main body of the paper in which a more general treatment of the problem is considered. It will be shown, in the coming sections, how it is possible to lump all the factors involved in vacuum tube amplifiers, i.e., physical circuit parameter and internal electronic effects involving the electron transit time, into a single coordinated picture with an equivalent circuit representation of the overall effect.

Equivalent Circuit Representation of Active Linear Four-Pole Pole Equations

Whenever tbe response of a general transducer is related in a linear manner to tbe stimulus, the transducer behavior is described by two linear relations. Although we are primarily concerned with electromagnetic transducers the concepts to be used are of broader utility and may, for example, also be applied to mechanical and electromechanical transducers.

Tbere are various ways in which tbe behavior of the four-pole may be expressed analytically. The form expressing current equilibrium bas already been given and it may well serve as a starting point for tbe following discussion:

Thus we have:

$$\left.\begin{aligned} I_1 &= \beta_{11}V_1 + \beta_{12}V_2 \\ I_2 &= \beta_{21}V_1 + \beta_{22}V_2 \end{aligned}\right\} \tag{14}$$

[2] F. B. Llewellyn, "Electron-Inertia Effects," Cambridge University Press, 1941.

[3] F. B. Llewellyn and L. C. Peterson, Interpretation of Ultra-High Frequency Tube Performance in Terms of Equivalent Networks, Proceedings of the National Electronics Conference, 1944.

as expressions for current equilibrium at any frequency. The four parameters β which appear have simple physical meanings, and it is seen that

β_{11} is the input admittance with output shorted

$-\beta_{22}$ is the output admittance with input shorted

$-\beta_{12}$ is the feedback admittance with input shorted

β_{21} is the transfer admittance with output shorted.

Before proceeding to the network representations of (14) it seems well to state briefly some of the reasons which almost force one to adopt the four-pole point of view when dealing with vacuum tubes in the higher frequency range.

There are the pedagogical reasons that classical methods long employed for passive networks are merely extended into the realm of active networks, thus providing unity in viewpoints.

The basic analysis involving the four-pole parameters for a particular transducer needs to be performed only once and, once obtained, all problems involving terminal impedances may, in any particular case, be solved in a routine manner.

There are also further practical reasons. We saw above that with increase in frequency the classical equivalent network had to be modified to a considerable extent in order to include the parasitic elements. This poses a serious problem for the tube designer, whose task it is to estimate the tube performance between known terminations. Such a task based upon the modified classical circuit becomes very difficult and cumbersome. Moreover, it is also difficult to segregate and measure the parasitic elements. Hence it appears that one could gain much if design parameters could be developed capable of reflecting parasitic and transit time effects. Finally, it is desirable to develop equivalent circuits with a minimum number of parameters bearing simple relationships to quantities which can be measured directly.

These general desires arising from the practical needs of the tube designer can be satisfied if tube behavior is specified by means of four-pole parameters.

All in all the four-pole point of view can be made to satisfy the logical needs of integrated concepts as well as the practical needs of simplicity in the specification of tube performance.

After this brief presentation of the argument for the four-pole point of view, the network representation of (14) will now be considered.

Stated in broadest terms: we are seeking a network representation by considering the two equations as a single unit and not by the trivial considera tion of each equation by itself.

We need, to begin with, the well-known network representation of a passive four-pole. Equation (14) has, then, the form

$$\left.\begin{aligned} I_1 &= \beta_{11}V_1 + \beta_{12}V_2 \\ I_2 &= -\beta_{12}V_1 + \beta_{22}V_2 \end{aligned}\right\} \tag{15}$$

and one equivalent circuit representation is that given by the Π network having the element values shown in Fig. 7. If so desired the Π network can of course also be transferred into an equivalent T network.

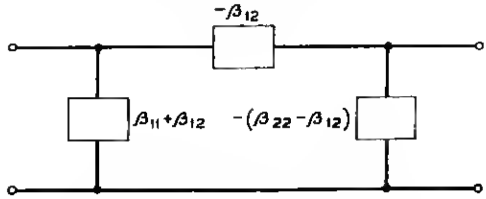


Fig. 7—Equivalent circuit of a passive four-pole.

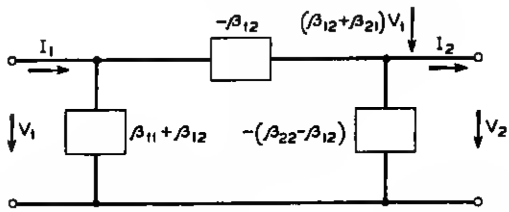


Fig. 8—Equivalent circuit of an active four-pole; current impressed at the output.

Now write (14) as

$$\left.\begin{aligned} I_1 &= \beta_{11}V_1 + \beta_{12}V_2 \\ I_2 &= -\beta_{12}V_1 + \beta_{22}V_2 + (\beta_{12} + \beta_{21})V_1 \end{aligned}\right\} \tag{16}$$

Whence it is seen by a comparison with (15) that the network representation of the active four-pole differs from the passive one merely by the presence of the impressed current $(\beta_{12} + \beta_{21})V_1$. A possible network representation of the general active four-pole is thus as shown on Fig. 8.

An immediate application may be illuminating. Consider, for example, the triode operated with positive grid, with interelectrode capacitances taken into account. The four-pole equations are given by (11) and (12) and the classical network is that of Fig. 5. From (12) and Fig. 8 we get the network of Fig. 9. It may be observed tbat, while in Fig. 5 the source and source-free constituents are intermingled, this is not the case in Fig. 9 where, on the contrary, a clear demarcation is present between such constituents.

In regard to the general network of Fig. 8 it may be noted that the network is composed of two parts. One part obeys the reciprocal law and is represented by a Π (or T) network and is consequently specified by three parameters. The other part is merely an impressed current controlled by the input potential V_1. From the fact that the Π (or T) network obeys the

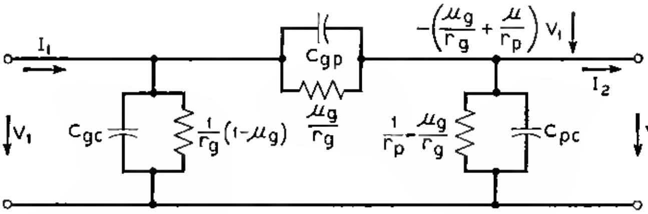


Fig. 9—Equivalent circuit of a positive grid triode at moderately low frequencies; current impressed at the output.

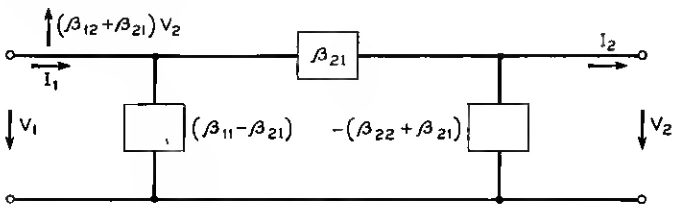


Fig. 10—Equivalent circuit of an active four-pole; current impressed at the input.

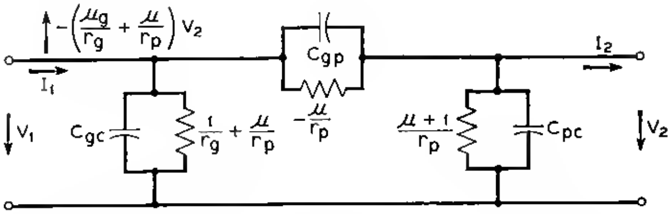


Fig. 11—Equivalent circuit of a positive grid triode at moderately low frequencies; current impressed at the input.

reciprocal law, the conclusion does not follow that it in general behaves as a passive network. The element values may, for example, be negative, and Bode's integral relations may not necessarily be true for this network.

It should further be noted that the network representation holds for all frequencies. The effect of frequency will be that the admittances in the network change and in changing they will, for example, reflect effects due to parasitics and electron transit time.

We next observe that the network of Fig. 8 is not unique; in other words it is not the only possihle network. To see this it need merely he observed that from the group of four independent β parameters in (15) there are several ways in which a suhgroup may be selected containing only three of them. For example the "passive part" of the network in Fig. 8 was constructed from the three parameters β_{11}, β_{12} and β_{22}. But this is evidently not the only choice. Moreover, the impressed force was taken to be voltage-controlled hut this again does not represent the only possihility.

In general it follows that the "passive part" of the network will reflect at least three properties of the complete network. In Fig. 8, for example, the "passive part" reproduces faithfully the two short-circuit driving-point admittances and the feedback admittance of the complete network.

Finally it is well to ohserve that only one driving force is needed in the general network formulation.

With this background of the general ideas involved let us now further explore some of the possibilities suggested as to other "passive network" constituents. Let us, for example, use β_{11}, β_{22} and β_{21} for the "passive part". We then write (14) as

$$\left.\begin{aligned} I_1 &= \beta_{11}V_1 - \beta_{21}V_2 + (\beta_{12} + \beta_{21})V_2 \\ I_2 &= \beta_{21}V_1 + \beta_{22}V_2 \end{aligned}\right\} \tag{17}$$

and from (17) the network representation of Fig. 10 follows. In addition this network differs from that of Fig. 8, in that the impressed current appears on the input side and that it is controlled by the output rather than by the input voltage. As an illustration consider again the triode operated with positive grid. The equivalent network is now as shown in Fig. 11.

Now let it be supposed that the impressed force be current rather than voltage-controlled. We then first transform (14) into

$$\left.\begin{aligned} V_1 &= \frac{I_1}{\beta_{11}} - \frac{\beta_{12}}{\beta_{11}} V_2 \\ I_2 &= \frac{\beta_{21}}{\beta_{11}} I_1 + V_2 \left(\beta_{22} - \frac{\beta_{21}\beta_{12}}{\beta_{11}}\right) \end{aligned}\right\} \tag{18}$$

and then rewrite (18) as

$$\left.\begin{aligned} V_1 &= \frac{I_1}{\beta_{11}} - \frac{\beta_{12}}{\beta_{11}} V_2 \\ I_2 &= -\frac{\beta_{12}}{\beta_{11}} I_1 + V_2 \left(\beta_{22} - \frac{\beta_{21}\beta_{12}}{\beta_{11}}\right) + \frac{\beta_{12} + \beta_{21}}{\beta_{11}} I_1 \end{aligned}\right\} \tag{19}$$

It can be shown that the network representation of Fig. 12 is consistent with (19). This network representation suffers from two obvious disadvantages. One is that the "passive network" is determined by only one short-circuit driving-point admittance of the complete network. The other short-circuit driving-point admittance of the "passive part" appears to be unrelated to any simple admittance which may be found from measurements at the output terminals of the complete network. The second disadvantage is that the coefficient of the current in the impressed force is of a complicated nature, since a driving-point admittance enters. Moreover, a closer investigation shows that in this network representation the "passive part" besides preserving the driving-point admittance β_{11} and the feedback admittance β_{12}, also preserves the quantity $\Delta_\beta = \beta_{11}\beta_{22} - \beta_{12}\beta_{21}$ of the complete network.

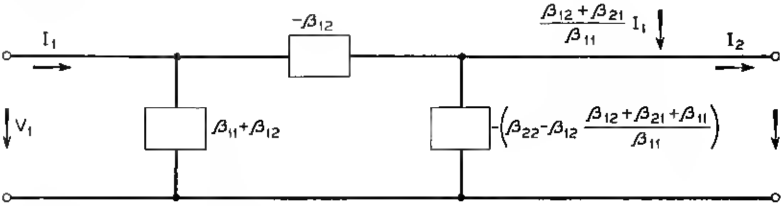


Fig. 12—Equivalent circuit of an active four-pole; current impressed at the output.

By proceeding from (19) in a way similar to that used in (17) the network in Fig. 12 transforms into another in which the impressed force appears on the input side. Many other networks can also be found.

The networks discussed were based on (14), which expressed the fact of current equilibrium and leads rather naturally to Π networks together with an impressed current source. On the other hand, starting with the four-pole equations which express voltage equilibrium, one encounters T networks together with impressed electromotive forces. These networks must now also be considered. In regard to the details involved in their derivation we may be very brief since the methods are similar to those already employed.

The four-pole equations expressing voltage equilibrium may be written as

$$\left.\begin{aligned} V_1 &= Z_{11}I_1 + Z_{12}I_2 \\ V_2 &= Z_{21}I_1 + Z_{22}I_2 \end{aligned}\right\} \tag{20}$$

where current and voltage directions are assumed to be taken in accordance with the conventions of Fig. 1.

The four parameters Z are of simple physical significance and it follows that:

Z_{11} is the input impedance with output open

$-Z_{22}$ is the output impedance with input open

$-Z_{12}$ is the feedback impedance with input open

Z_{21} is the transfer impedance with output open.

The relations between the β's and the Z's are given by the expressions:

$$\left.\begin{aligned} \beta_{11} &= \frac{Z_{22}}{\Delta_Z} \\ \beta_{12} &= -\frac{Z_{12}}{\Delta_Z} \\ \beta_{21} &= -\frac{Z_{21}}{\Delta_Z} \\ \beta_{22} &= \frac{Z_{11}}{\Delta_Z} \end{aligned}\right\} \tag{21}$$

where

$$\Delta_Z = \begin{vmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{vmatrix} \tag{22}$$

We have also

$$\Delta_Z = \frac{1}{\Delta_\beta} \tag{23}$$

where

$$\Delta_\beta = \begin{vmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{vmatrix} \tag{24}$$

Applying now to (20) the transformations which led to the networks of Figs. 9, 10 and 12, we get the networks of Figs. 13, 14 and 15.

The "passive part" of the network in Fig. 13 reproduces the two open circuit impedances Z_{11} and Z_{22} as well as the feedback impedance Z_{12} of the complete network.

The network in Fig. 14 differs from that of Fig. 13 because of the use of the transfer impedance Z_{21} in the "passive part" with the result that the impressed current appears on the input side.

The "passive part" in Fig. 15, finally, preserves the open-circuit impedance Z_{11}, the feedback impedance Z_{12} and the determinant Δ_Z of the complete network. This network shows incidentally a close resemblance to one already published.[3]

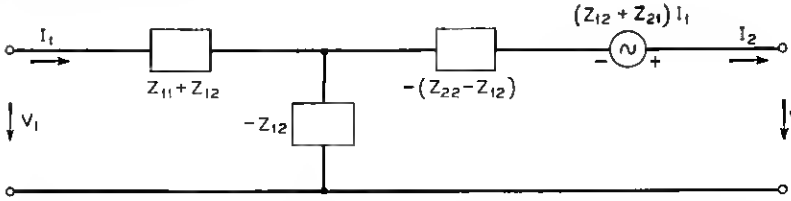


Fig. 13—Equivalent circuit of an active four-pole; voltage impressed in series with the output.

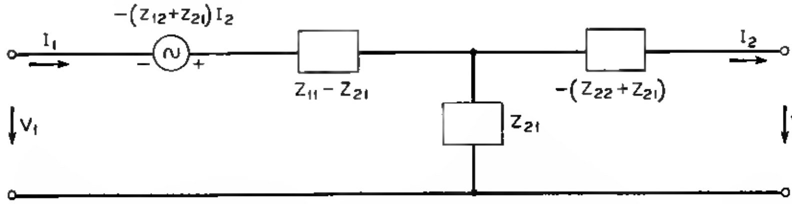


Fig. 14—Equivalent circuit of an active four-pole; voltage impressed in series with the input.

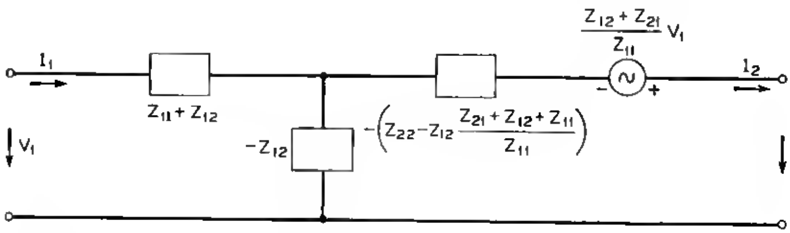


Fig. 15—Equivalent circuit of an active four-pole; voltage impressed in series with the output.

It is well to emphasize that, while all the complete networks are equivalent, this is not true for the "passive parts." In fact from their derivation it follows that none are equivalent. Specific circumstances may make it desirable to perform transformations on the passive parts. For example, it might be more convenient to work with a Π than with a T network. Such transformations are of course perfectly legitimate and raise the question of choice of equivalent networks. A few remarks on this subject may be

[3] Loc. cit.

appropriate, since the choice is not quite a matter of indifference. Broadly speaking, the choice will depend upon the relative advantages of nodal and mesh analysis and in most practical situations the former has proved to be the more convenient of the two. One cannot, however, be too dogmatic in this regard. Consider the networks of Figs. 8 and 13. Suppose, for example, that parasitic elements appearing as a passive Π network had to be superimposed. It is then more conventient to use Fig. 8, not only on account of the ease with which this may be done, but also on account of the fact that the effective transadmittance $\beta_{12} + \beta_{21}$ is invariant with respect to such a superposition. If, on the other hand, parasitic elements appear as series elements (lead inductances for example) the network of Fig. 13 might be more convenient since the effective transimpedance $Z_{12} + Z_{21}$ now remains invariant.

It is also desirable to choose an equivalent network whose elements are capable of being determined by simple measurements; from this consideration the network on Fig. 8 is of distinct advantage.

Application to Triodes

The preceding section was primarily directed towards the development of possible forms of network representations of the general four-pole equations. In this section one of these forms, namely that given in Fig. 8, will be used to represent the three modes of triode operation. Depending upon which electrode is at a-c ground potential, we may distinguish between the following methods of operation:

1. Grounded cathode operation.
2. Grounded grid operation.
3. Grounded plate operation.

The schematic diagrams, together with assumed voltage and current directions for these modes, are shown on Figs. 16, 17 and 18 respectively.

With a given set of available terminals the first step in obtaining the networks consists in calculating the four-pole parameters with respect to these terminals. It will be assumed that the coupling circuits have been designed with such efficiency that lead effects can be disregarded, so that the available terminals actually coincide with anode, grid and cathode. This set of available terminals brings us as close to the electron stream as it is physically possible to attain and it represents the ideal towards which design tends.

It is beyond the scope of this paper to consider the details involved in the calculations of the four-pole parameters. The basic tools needed are the result of a study of the dynamics of the electron stream, which started from

fundamentals,[4] and some familiarity with this work is assumed on the part of the reader. Concerning these tools two reservations need be made. In the first place the tools apply to planar rather than to cylindrical structures. Since, however, there is a decided tendency toward planar structures, especially in the higb-frequency field, because of a desire for uniform electron streams, this limitation does not seem serious. In tbe second place the tools are also subject to the limitation of a single-valued velocity electron stream.

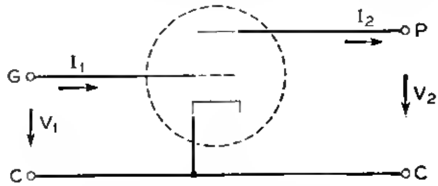


Fig. 16—Vurrent-voltage relations for the grounded cathode triode.

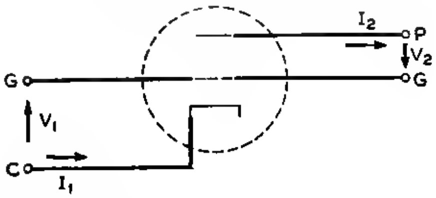


Fig. 17—Current-voltage relations for the grounded grid triode.

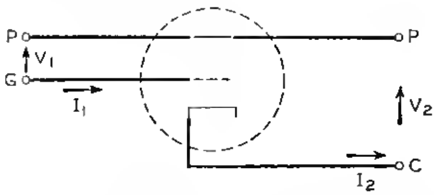


Fig. 18—Current-voltage relations for the grounded plate triode.

This, again, is not too serious, since one of the aims in present high-frequency tube design is to produce as uniform a stream as possible. Nevertheless, the effects produced by multiple velocities are important to know. Studies along such lines have been made by Mr. Frank Gray of these Laboratories.

The operating conditions of tbe triode are assumed to be quite general. There are, for example, no restrictions placed upon frequency and space charge and the grid may, moreover, have either positive or negative d-c potential with respect to the catbode.

[4] F. B. Llewellyn and L. C. Peterson, "Vacuum Tube Networks," Proceedings of the I.R.E., March, 1944.

With current and voltage directions as in Figs. 16, 17 and 18, the following Tables I, II and III list the four-pole parameters for the three modes of triode operation in both β and Z forms.

TABLE I

FOUR-POLE PARAMETERS FOR GROUNDED CATHODE TRIODE

$$\left.\begin{aligned}
\beta_{11} &= \frac{y_{11} + y_{21} + y_{22}}{D} \qquad \beta_{12} = -\frac{y_{22}}{D} \\
\beta_{21} &= \frac{y_{21} + y_{22}}{D} \qquad \beta_{22} = -\frac{y_{22} + \dfrac{y_{11}}{\mu}}{D} \\
\Delta_\beta &= \begin{vmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{vmatrix} = -\frac{y_{11}\, y_{22}}{D} \\
D &= 1 + \frac{y_{11} + y_{21} + y_{22}}{\mu y_{22}}
\end{aligned}\right\}$$

$$\left.\begin{aligned}
Z_{11} &= \frac{1}{y_{11}} + \frac{1}{\mu y_{22}} \qquad Z_{12} = -\frac{1}{y_{11}} \\
Z_{21} &= \frac{1}{y_{11}} + \frac{y_{21}}{y_{11}\, y_{22}} \qquad Z_{22} = -\left[\frac{1}{y_{11}} + \frac{1}{y_{22}} + \frac{y_{21}}{y_{11}\, y_{22}}\right] \\
\Delta_Z &= \begin{vmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{vmatrix} = \frac{1}{\Delta_\beta} = -\frac{D}{y_{11}\, y_{22}}
\end{aligned}\right\}$$

μ=amplication factor.

The y admittance coefficients appearing in the above tables were fully explained and discussed in the paper on Vacuum Tube Networks, to which reference has already been made. Suffice it here to say that y_{11} is the admittance of the diode coinciding with cathode and equivalent grid plane and y_{22} the admittance of the diode coinciding with the equivalent grid plane and the anode and finally y_{21} the transadmittance between these fictitious diodes. The admittance y_{11} depends upon the d-c conditions between cathode and grid and upon the transit angle for this region alone. The diode admittance y_{22} depends in a similar manner upon the d-c space charge conditions in the grid-anode region as well as upon the transit angle for this region alone. For the small degree of space charge which usually exists between grid and plate of most triodes, y_{22} can be represented by a simple capacitance. The transadmittance y_{21} can be resolved into two factors, the first of which depends only upon the transit angle between cathode and grid, and the second only upon the transit angle between grid and anode. In the paper on vacuum tube networks all these admittances were plotted

TABLE II

FOUR-POLE PARAMETERS FOR GROUNDED GRID TRIODE

$$\left.\begin{aligned}
\beta_{11} &= \frac{y_{11}\left(1+\frac{1}{\mu}\right)}{D} \qquad \beta_{12} = -\frac{y_{11}}{\mu D} \\
\beta_{21} &= -\frac{y_{21}-\frac{y_{11}}{\mu}}{D} \qquad \beta_{22} = -\frac{y_{22}+\frac{y_{11}}{\mu}}{D} \\
\Delta_\beta &= \begin{vmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{vmatrix} = -\frac{y_{11}\,y_{22}}{D} \\
D &= 1 + \frac{y_{11}+y_{21}+y_{22}}{\mu y_{22}}
\end{aligned}\right\}$$

$$\left.\begin{aligned}
Z_{11} &= \frac{1}{y_{11}} + \frac{1}{\mu y_{22}} \qquad Z_{12} = -\frac{1}{\mu y_{22}} \\
Z_{21} &= \frac{1}{\mu y_{22}} - \frac{y_{22}}{y_{11}\,y_{22}} \qquad Z_{22} = -\left[\frac{1}{y_{22}} + \frac{1}{\mu y_{22}}\right] \\
\Delta_Z &= \begin{vmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{vmatrix} = \frac{1}{\Delta_\beta} = -\frac{D}{y_{11}\,y_{22}}
\end{aligned}\right\}$$

μ = amplification factor.

TABLE III

FOUR-POLE PARAMETERS FOR GROUNDED PLATE TRIODE

$$\left.\begin{aligned}
\beta_{11} &= \frac{y_{11}+y_{21}+y_{22}}{D} \qquad \beta_{12} = -\frac{y_{11}+y_{21}}{D} \\
\beta_{21} &= \frac{y_{11}}{D} \qquad \beta_{22} = -\frac{y_{11}\left(1+\frac{1}{\mu}\right)}{D} \\
\Delta_\beta &= \begin{vmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{vmatrix} = -\frac{y_{11}\,y_{22}}{D} \\
D &= 1 + \frac{y_{11}+y_{21}+y_{22}}{\mu y_{22}}
\end{aligned}\right\}$$

$$\left.\begin{aligned}
Z_{11} &= \frac{1}{y_{22}} + \frac{1}{\mu y_{22}} \qquad Z_{12} = -\left[\frac{1}{y_{22}} + \frac{y_{21}}{y_{11}\,y_{22}}\right] \\
Z_{21} &= \frac{1}{y_{22}} \qquad Z_{22} = -\left[\frac{1}{y_{11}} + \frac{1}{y_{22}} + \frac{y_{12}}{y_{11}\,y_{22}}\right] \\
\Delta_Z &= \begin{vmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{vmatrix} = \frac{1}{\Delta_\beta} = -\frac{D}{y_{11}\,y_{22}}
\end{aligned}\right\}$$

μ = amplication factor.

graphically, showing both phase and magnitude, and this paper is referred to for details.[4]

Tables I, II and III, in conjunction with Figs. 8, 13 and 15, allow us to derive the equivalent networks of Figs. 19, 20 and 21.

We must now undertake a discussion of the results given in the tables as well as of the networks which were derived from them.

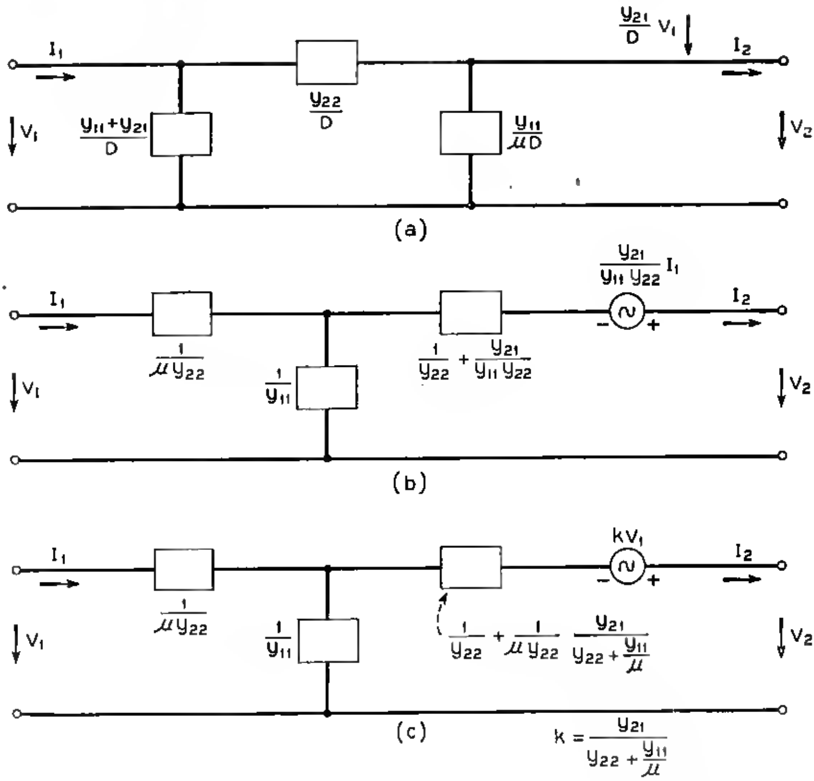


Fig. 19 (a, b, c)—Three forms of equivalent circuits of the grounded cathode triode valid at all frequencies.

Initially, it is well to emphasize again that the four-pole parameters and the corresponding networks are different but equivalent ways through which the triode signal behavior becomes completely specified for all conditions of space charge and for all frequencies.

Secondly, there are certain general relations which should be noted. We observe, first, that the determinants Δ_β and Δ_Z are invariants for the different modes of triode operation, and with exception of phase reversals this

[4] Loc. cit.

is also the case for the effective transadmittance $\beta_{12} + \beta_{21}$ as well as for the effective transimpedance $Z_{12} + Z_{21}$. On the other hand, the quantity $\frac{Z_{12} + Z_{21}}{Z_{11}}$, which appears in the network of Fig. 15 and which represents the driving force per unit input voltage, is invariant (except for reversal in phase) only for grounded grid and grounded cathode operation. Several

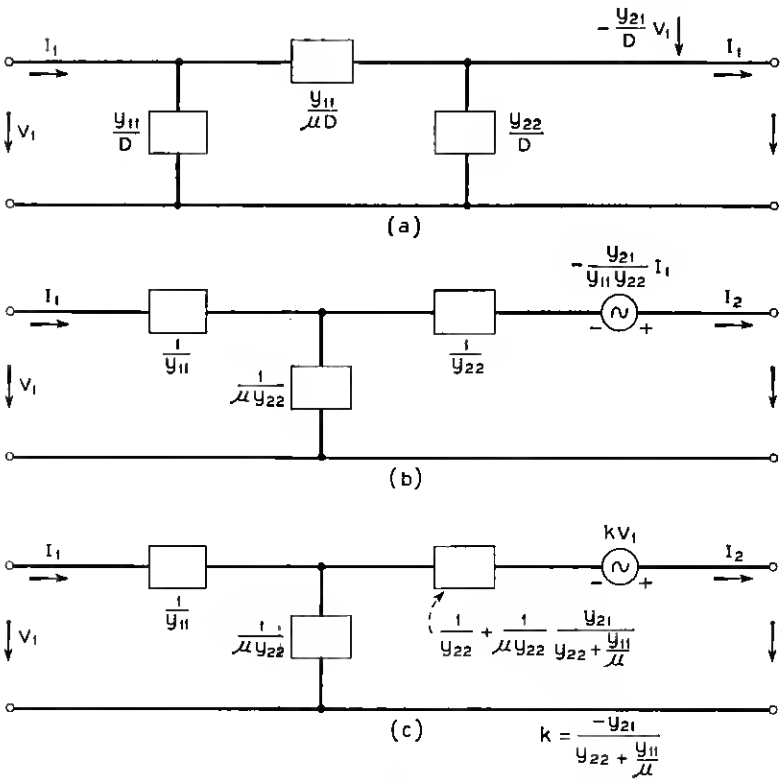


Fig. 20 (a, b, c)—Three forms of equivalent circuits of the grounded grid triode valid at all frequencies.

admittance and impedance relations may also be pointed out. For example, the input short-circuit driving-point admittance β_{11} is equal for grounded cathode and grounded plate operation and the same is true for the output short-circuit driving-point admittances β_{22} for grounded cathode and grounded grid operation. Moreover, it is also seen that the input short-circuit driving-point admittance β_{11} for grounded grid operation is equal to the output driving-point admittance β_{22} for grounded plate operation. A

similar set of reciprocal relations between the open-circuit driving-point impedances is also present.

In regard to the networks it may first be observed that, since they were derived from parameters upon which no restrictions had been placed on either frequency or space charge, they are also generally valid. In passive circuit theory one is accustomed to the use of only the three basic elements

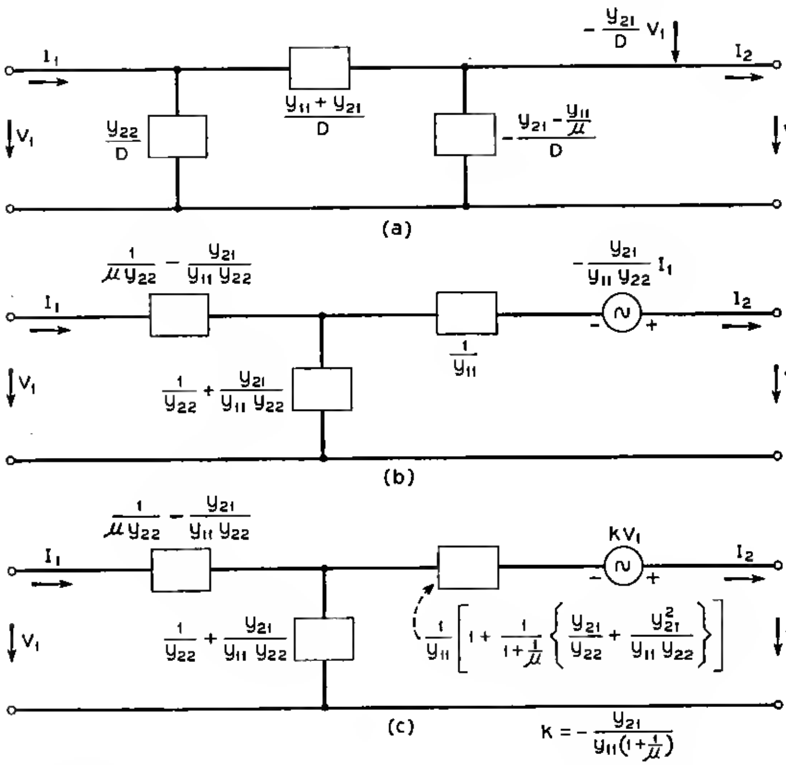


Fig. 21 (a, b, c)—Three forms of equivalent circuits of the grounded plate triode valid at all frequencies.

of resistance, inductance and capacitance. For the networks now under consideration other quantities also need to be included. However, as normally operated there is usually complete space-charge in the cathode-grid region and a very small amount of space charge in the grid-plate region. Under such circumstances the admittance y_{22} is a simple capacitance and the amplification factor μ is a real number. The admittances y_{11} and y_{21} , on the other hand, do not allow accepted circuit interpretation to be made, except in the range of moderately low frequencies when electron transit

time is taken into account only to a first order of approximation. It is believed that, in general, these admittances should be considered complete by themselves as new admittance elements, and, as already remarked, their values in magnitude and phase may be found in the paper on vacuum tube networks to which repeated reference has been made.

As a further property of the networks, consider the expressions for the β's in Tables I, II and III. It is observed that each β_{ij} of a set of β's contains a common term, suggesting that the networks might be broken up into at least two elementary constituents. The same observation applies to each of the three sets of Z's. In Table I, for example, it is seen that this constant term is represented by y_{22}/D. The network of Fig. 19a can thus be thought of as arising from the superposition of two networks, one of which is determined by the parameters

$$\left.\begin{aligned} \beta'_{11} &= \frac{y_{11}+y_{21}}{D} \qquad & \beta'_{12} &= 0 \\ \beta'_{21} &= \frac{y_{21}}{D} \qquad & \beta'_{22} &= -\frac{y_{11}}{\mu D} \end{aligned}\right\} \tag{25}$$

and the other by the parameters

$$\left.\begin{aligned} \beta''_{11} &= \frac{y_{22}}{D} \qquad & \beta''_{12} &= -\frac{y_{22}}{D} \\ \beta''_{21} &= \frac{y_{22}}{D} \qquad & \beta''_{22} &= -\frac{y_{22}}{D} \end{aligned}\right\} \tag{26}$$

The admittance coefficients given by (25) correspond to the perfectly unilateral active network shown in Fig. 22a, while the admittance coefficients in (26) correspond to the "passive" network in Fig. 22b. The two elementary constituents thus take the general forms of these networks. It should be noticed that these two network constituents are unrelated to the fact that the total current entering the complete network is the sum of conduction and displacement current.

Corresponding to the Z's other elementary constituents are obtained, with general forms as shown on Figs. 23a and 23b.

These elementary constituents are merely reflections of certain mathematical identities. For example, the networks in Fig. 22 depend upon the matrix identity

$$\begin{Vmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{Vmatrix} = \begin{Vmatrix} \beta_{11}+\beta_{12} & 0 \\ \beta_{21}+\beta_{12} & \beta_{22}-\beta_{12} \end{Vmatrix} + \begin{Vmatrix} -\beta_{12} & \beta_{12} \\ -\beta_{12} & \beta_{12} \end{Vmatrix} \tag{27}$$

while those in Fig. 23 depend upon a corresponding identity. The identity (27) expresses the fact that the general "β-network" can be considered as arising from the parallel connections of the two networks in Fig. 24.

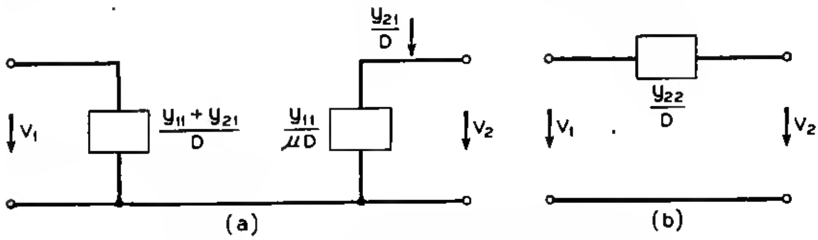


Fig. 22 (a, b)—Elementary constituents of Fig. 19a.

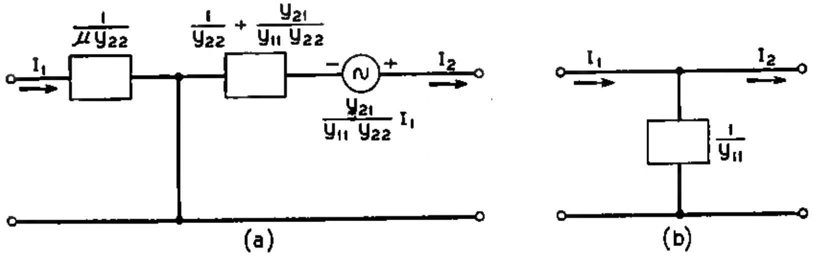


Fig. 23 (a, b)—Elementary constituents of Fig. 19b.

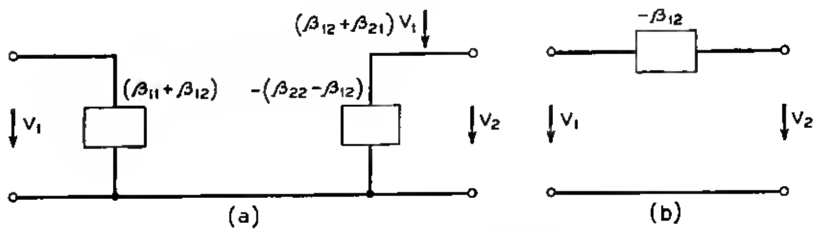


Fig. 24 (a, b)—Elementary constituents of Fig. 8.

There is at present a tendency towards grid designs of very fine mesh. Such a grid design results in a very large value of the amplification factor and, for many purposes, sufficient accuracy may be obtained by disregarding terms containing $\frac{1}{\mu}$ as a factor. Under these conditions the network for grounded cathode operation reduces to an L-network, that for grounded grid operation to a unilateral network transmitting in the direction from grid to plate only, while the cathode follower network remains essentially unchanged.

Table IV

Four-Pole Parameters for Grounded Cathode Triode in the Range of Moderately Low Frequencies

$$\beta_{11} = \frac{1}{5}\frac{(\omega C_1)^2}{g_0}\frac{F}{\left[1+\frac{1}{\mu}\left(1+\frac{4}{3}\frac{x_2}{x_1}f\right)\right]^2} + i\omega\frac{\frac{4}{3}C_1 f + C_2}{1+\frac{1}{\mu}\left(1+\frac{4}{3}\frac{x_2}{x_1}f\right)}$$

$$\beta_{12} = -\left[-\frac{(\omega C_1)^2}{g_0}\frac{1}{5\mu}\frac{F}{\left[1+\frac{1}{\mu}\left(1+\frac{4}{3}\frac{x_2}{x_1}f\right)\right]^2} + i\omega\frac{C_2}{1+\frac{1}{\mu}\left(1+\frac{4}{3}\frac{x_2}{x_1}f\right)}\right]$$

$$\beta_{22} = -\left[\frac{g_0}{\mu}\frac{1}{1+\frac{1}{\mu}\left(1+\frac{4}{3}\frac{x_2}{x_1}f\right)} + i\omega\frac{C_2+\frac{6}{10\mu}C_1}{1+\frac{1}{\mu}\left(1+\frac{4}{3}\frac{x_2}{x_1}f\right)}\right]$$

$$\beta_{12}+\beta_{21} = -\frac{g_0}{1+\frac{1}{\mu}\left(1+\frac{4}{3}\frac{x_2}{x_1}f\right)}$$

$$\left[1 - i\frac{11}{30}\theta_1\left(1-\frac{3}{11\mu}\frac{\frac{x_2}{x_1}F}{1+\frac{1}{\mu}\left(1+\frac{4}{3}\frac{x_2}{x_1}f\right)}\right) - i\frac{1}{3}\theta_2 k\right]$$

Table V

Four-Pole Parameters for Grounded Grid Triode in the Range of Moderately Low Frequencies

$$\beta_{11} = g_0\frac{1+\frac{1}{\mu}}{1+\frac{1}{\mu}\left(1+\frac{4}{3}\frac{x_2}{x_1}f\right)} + i\omega\frac{6}{10}C_1\frac{1+\frac{1}{\mu}}{1+\frac{1}{\mu}\left(1+\frac{4}{3}\frac{x_2}{x_1}f\right)}\left[1+\frac{\frac{1}{\mu}\frac{x_2}{x_1}}{1+\frac{1}{\mu}\left(1+\frac{4}{3}\frac{x_2}{x_1}f\right)}\right]$$

$$\beta_{12} = -\frac{g_0}{\mu}\cdot\frac{1}{1+\frac{1}{\mu}\left(1+\frac{4}{3}\frac{x_2}{x_1}f\right)} - i\omega\frac{6}{10}\frac{C_1}{\mu}$$

$$\cdot\frac{1}{1+\frac{1}{\mu}\left(1+\frac{4}{3}\frac{x_2}{x_1}f\right)}\left[1+\frac{1}{3}\frac{\frac{1}{\mu}\frac{x_2}{x_1}F}{1+\frac{1}{\mu}\left(1+\frac{4}{3}\frac{x_2}{x_1}f\right)}\right]$$

$$\beta_{22} = -\left[\frac{g_0}{\mu}\cdot\frac{1}{1+\frac{1}{\mu}\left(1+\frac{4}{3}\frac{x_2}{x_1}f\right)} + i\omega\left(\frac{C_2+\frac{6}{10\mu}C_1}{1+\frac{1}{\mu}\left(1+\frac{4}{3}\frac{x_2}{x_1}f\right)}\right)\right]$$

$$\beta_{12}+\beta_{21} = \frac{g_0}{1+\frac{1}{\mu}\left(1+\frac{4}{3}\frac{x_2}{x_1}f\right)}\left[1 - i\frac{11}{30}\theta_1\left(1-\frac{3}{11\mu}\frac{\frac{x_2}{x_1}F}{1+\frac{1}{\mu}\left(1+\frac{4}{3}\frac{x_2}{x_1}f\right)}\right) - i\frac{1}{3}\theta_2 k\right]$$

In the foregoing, the behavior of an active four-pole has been described either in terms of four-pole parameters or in terms of elements of an equivalent circuit. The particular four-pole parameters, which are of customary use in communication engineering, are the so-called image parameters, but they have usually been used only in connection with passive four-poles. They may, however, also be used in the more general vacuum tube four-pole now under discussion, but whether their employment would be of practical

TABLE VI

FOUR-POLE PARAMETERS FOR GROUNDED PLATE TRIODE IN THE RANGE OF MODERATELY LOW FREQUENCIES

$$\beta_{11} = \frac{1}{5}\frac{(\omega C_1)^2}{g_0}\frac{F}{\left[1+\frac{1}{\mu}\left(1+\frac{4}{3}\frac{x_2}{x_1}f\right)\right]^2} + i\omega\frac{\frac{4}{3}C_1 f + C_2}{1+\frac{1}{\mu}\left(1+\frac{4}{3}\frac{x_2}{x_1}f\right)}$$

$$\beta_{12} = -\left[\frac{1}{5}\frac{(\omega C_1)^2}{g_0}\frac{F\left(1+\frac{1}{\mu}\right)}{\left[1+\frac{1}{\mu}\left(1+\frac{4}{3}\frac{x_2}{x_1}f\right)\right]^2} + i\omega\frac{\frac{4}{3}C_1 f}{1+\frac{1}{\mu}\left(1+\frac{4}{3}\frac{x_2}{x_1}f\right)}\right]$$

$$\beta_{22} = -\left[g_0\frac{1+\frac{1}{\mu}}{1+\frac{1}{\mu}\left(1+\frac{4}{3}\frac{x_2}{x_1}f\right)} + i\omega\frac{6}{10}C_1\frac{1+\frac{1}{\mu}}{1+\frac{1}{\mu}\left(1+\frac{4}{3}\frac{x_2}{x_1}f\right)} \cdot \left[1+\frac{1}{3}\frac{\frac{1}{\mu}\frac{x_2}{x_1}F}{1+\frac{1}{\mu}\left(1+\frac{4}{3}\frac{x_2}{x_1}f\right)}\right]\right]$$

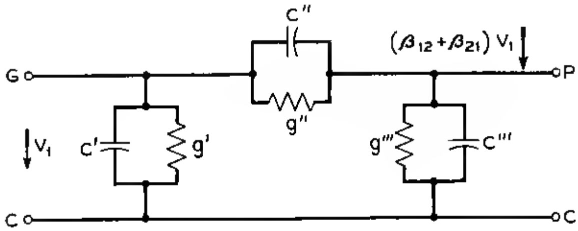
$$\beta_{12}+\beta_{21} = \frac{g_0}{1+\frac{1}{\mu}\left(1+\frac{4}{3}\frac{x_2}{x_1}f\right)}\left[1 - i\frac{11}{30}\theta_1\left(1-\frac{3}{11\mu}\frac{\frac{x_2}{x_1}F}{1+\frac{1}{\mu}\left(1+\frac{4}{3}\frac{x_2}{x_1}f\right)}\right) - i\frac{1}{3}\theta_2 k\right]$$

value is a matter which engineering experience will decide. In any case their use would be limited to vacuum tubes in which appreciable interaction between input and output terminals is present. From a practical standpoint this means that their usefulness would be mainly found in connection with triodes.

TRIODE NETWORKS AT MODERATELY LOW FREQUENCIES

The networks discussed in the preceding section were of general validity in respect to both operating conditions and frequency and it was mentioned

that the efforts of trying to interpret the "passive" parts of the networks in the form of lumped passive circuit elements had, in general, met with not too much success. In this section attention will be given to the range of moderately low frequencies where usual circuit interpretation is possible. The operating conditions are assumed to be the usual ones with complete



$$g' = \frac{1}{5}\frac{(\omega C_1)^2}{g_0} F \frac{1+\frac{1}{\mu}}{\left[1+\frac{1}{\mu}\left(1+\frac{4}{3}\frac{x_2}{x_1}f\right)\right]^2}$$

$$C' = \frac{4}{3} C_1 \frac{f}{1+\frac{1}{\mu}\left(1+\frac{4}{3}\frac{x_2}{x_1}f\right)}$$

$$g'' = -\frac{(\omega C_1)^2}{5g_0}\frac{1}{\mu}\frac{F}{1+\frac{1}{\mu}\left(1+\frac{4}{3}\frac{x_2}{x_1}f\right)}$$

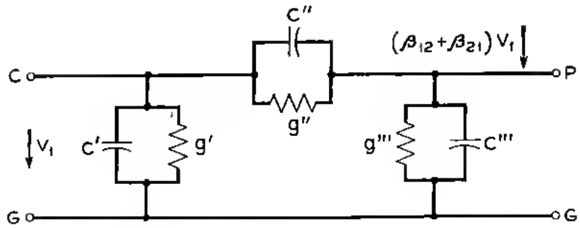
$$C'' = C_2 \frac{1}{1+\frac{1}{\mu}\left(1+\frac{4}{3}\frac{x_2}{x_1}f\right)}$$

$$g''' = \frac{g_0}{\mu}\frac{1}{1+\frac{1}{\mu}\left(1+\frac{4}{3}\frac{x_2}{x_1}f\right)}$$

Fig. 25—Equivalent circuit of grounded cathode triode at moderately high frequencies.

space charge in the cathode-grid region, and negligible space charge in the grid-plate region. Also it will be assumed that the grid is at negative d-c potential with respect to the cathode.

The first step is to expand the β coefficients in series with transit angles retained only to first or possibly to second orders. As the detailed computations are lengthy only the final result will be given. These are presented in Tables IV, V and VI.



$$g' = g_0 \frac{1}{1 + \frac{1}{\mu}\left(1 + \frac{4}{3}\frac{x_2}{x_1} f\right)}$$

$$C' = \frac{6}{10} C_1 \frac{1}{1 + \frac{1}{\mu}\left(1 + \frac{4}{3}\frac{x_2}{x_1} f\right)} \left[1 + \frac{1}{3\mu} \frac{\frac{x_2}{x_1} F}{1 + \frac{1}{\mu}\left(1 + \frac{4}{3}\frac{x_2}{x_1} f\right)} \right]$$

$$g'' = \frac{1}{\mu} g'$$

$$C'' = \frac{1}{\mu} C'$$

$$g''' = -\frac{(\omega C_1)^2}{5 g_0} \frac{1}{\mu} \frac{F}{\left[1 + \frac{1}{\mu}\left(1 + \frac{4}{3}\frac{x_2}{x_1} f\right)\right]^2}$$

$$C''' = C_2 \frac{1}{1 + \frac{1}{\mu}\left(1 + \frac{4}{3}\frac{x_2}{x_1} f\right)}$$

Fig. 26—Equivalent circuit of grounded grid triode at moderately high frequencies.

In these tables the symbols have the following meanings:

g_0 = static conductance of the diode formed by the cathode and equivalent grid-plane.

μ = low frequency amplification factor.

x_1 = cathode-grid distance in cm.

x_2 = grid-anode distance in cm.

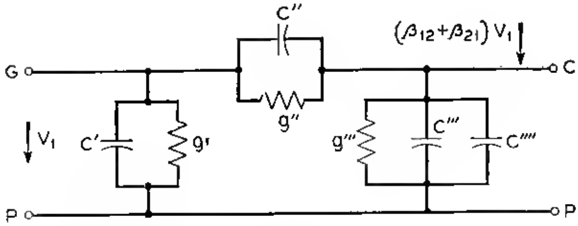
C_1 = cold capacitance between cathode and equivalent grid plane

C_2 = cold capacitance between anode and equivalent grid plane.

Θ_1 = electron transit angle between cathode and equivalent grid plane. It is most simply calculated from

$$\theta_1 = 2\frac{\omega C_1}{g_0}$$

Θ_2 = electron transit angle between anode and equivalent grid plane.



$$g' = -\frac{(\omega C_1)^2}{5g_0}\frac{1}{\mu}\frac{F}{\left[1+\frac{1}{\mu}\left(1+\frac{4}{3}\frac{x_2}{x_1}f\right)\right]^2}$$

$$C' = C_2\frac{1}{1+\frac{1}{\mu}\left(1+\frac{4}{3}\frac{x_2}{x_1}f\right)}$$

$$g'' = \frac{(\omega C_1)^2}{5g_0}F\frac{1+\frac{1}{\mu}}{\left[1+\frac{1}{\mu}\left(1+\frac{4}{3}\frac{x_2}{x_1}f\right)\right]^2}$$

$$C'' = \frac{4}{3}C_1\frac{f}{1+\frac{1}{\mu}\left(1+\frac{4}{3}\frac{x_2}{x_1}f\right)}$$

$$g''' = g_0\frac{1+\frac{1}{\mu}}{1+\frac{1}{\mu}\left(1+\frac{4}{3}\frac{x_2}{x_1}f\right)}$$

$$C''' = \frac{6}{10}\frac{C_1}{\mu}\frac{1}{1+\frac{1}{\mu}\left(1+\frac{4}{3}\frac{x_2}{x_1}f\right)}\left[1+\frac{1}{3}\frac{\frac{x_2}{x_1}F}{1+\frac{1}{\mu}\left(1+\frac{4}{3}\frac{x_2}{x_1}f\right)}\right]$$

$$C'''' = -\frac{11}{15}C_1\frac{1+\frac{10}{11}\frac{\theta_2}{\theta_1}k_1}{1+\frac{1}{\mu}\left(1+\frac{4}{3}\frac{x_2}{x_1}f\right)}$$

Fig. 27—Equivalent circuit of grounded plate triode at moderately high frequencies.

It may be calculated from

$$\theta_2 = \frac{\omega 2 x_2}{\sqrt{2\eta}\,(\sqrt{V_{D1}} + \sqrt{V_{D2}})}$$

where $\eta = 10^7 \dfrac{e}{m} = 1.76 \times 10^{15}$

V_{D1} = equivalent d-c grid potential

V_{D2} = anode d-c potential

$$F = 1 + \frac{22}{9}\frac{\theta_2}{\theta_1}\frac{\sqrt{V_{D1}} + 2\sqrt{V_{D2}}}{\sqrt{V_{D1}} + \sqrt{V_{D2}}} + \frac{5}{3}\left(\frac{\theta_2}{\theta_1}\right)^2 \frac{\sqrt{V_{D1}} + 3\sqrt{V_{D2}}}{\sqrt{V_{D1}} + \sqrt{V_{D2}}}$$

$$f = 1 + \frac{1}{2}\frac{\theta_2}{\theta_1}\frac{\sqrt{V_{D1}} + 2\sqrt{V_{D2}}}{\sqrt{V_{D1}} + \sqrt{V_{D2}}}$$

$$k = \frac{\sqrt{V_{D1}} + 2\sqrt{V_{D2}}}{\sqrt{V_{D1}} + \sqrt{V_{D2}}}$$

With the aid of these tables and Fig. 8 the equivalent circuits on Figs. 25, 26 and 27 are obtained. The networks are all of the resistance-capacity type. It may be noted that, in some of the branches, negative conductance or negative capacitance appears. However, as seen from the external tube terminals they are swamped by corresponding positive elements.

The viewpoints presented in this paper have been used by the writer over a number of years. They have been given experimental application by Mr. J. A. Morton, who is principally responsible for their introduction and use in the studies in these Laboratories of electron tubes in the microwave regions.

With this, our investigation comes to a close. Much has been omitted, particularly in the field of applications, but it is nevertheless hoped the fundamental approach, as well as the networks given, may prove to be useful in practical applications. The questions of noise and of optimum noise figure design have also been left out of consideration. Mr. J. A. Morton and the writer plan to discuss these problems in a forthcoming paper.

The writer is pleased to acknowledge his indebtedness to Messrs. R. K. Potter, J. A. Morton, and R. M. Ryder, who have encouraged this work and urged its publication; and to Mr. W. E. Kirkpatrick for constructively critical scrutiny of the original technical memorandum.